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Localized states in triplet superconductor–ferromagnet–triplet superconductor junctions

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Abstract

We examine the localized Andreev states formed at a triplet superconductor–ferromagnet–triplet superconductor Josephson junction. These mid-gap states dominate the low-energy transport through the junction, which shows a rich dependence upon the junction parameters. We compare and contrast the characteristics of the Josephson current for a ferromagnetic moment aligned parallel and perpendicular to the junction interface. We find that spin-polarized currents are possible, as well as an unusual temperature dependence of the Josephson current.

(Some figures in this article are in colour only in the electronic version)

The construction of novel Josephson junctions using unconventional superconductors or barriers has recently attracted much interest [1–7]. An important aspect of these investigations concerns the formation of mid-gap Andreev bound states, which are localized at the junction barrier. Due to the breakdown of translational invariance at the junction these states typically differ considerably from the bulk quasiparticle states, for example they can display additional pairing-state symmetries or odd-frequency pairing [8, 9]. The key experimental signature of the mid-gap Andreev states is the resonant tunneling through them, which is responsible for the low-temperature anomaly in the Josephson current observed in unconventional junctions [1, 3].

In this paper we investigate an unconventional triplet superconductor–ferromagnet–triplet superconductor (TFT) Josephson junction, which is schematically illustrated in figure 1. This junction geometry was first introduced in [7]. We treat the insulating ferromagnetic barrier within the δ-function approximation, which is reasonable if its width is much smaller than the coherence length in the bulk superconductors [10]. Similarly, although the pairing amplitude should be suppressed near to the barrier due to the proximity effect, the order parameter recovers its bulk value on a length scale much smaller than the decay length of the Andreev bound state so we assume that the pairing amplitude is uniform within each superconducting slab [10]. We do not expect our results to be qualitatively altered by relaxing these commonly adopted assumptions. We extend the results of [7] by comparing and contrasting the effect of a ferromagnetic moment perpendicular and parallel to the junction interface. We find that these two cases display qualitatively different dependence upon the junction parameters. Of particular note is the Josephson current reversal with temperature when the magnetic moment is perpendicular to the junction; for a magnetic moment parallel to the junction interface, we find a spin-polarized Josephson current when the **d** vectors of the left and right superconductors are not aligned.

We assume that the barrier lies along the plane $z = 0$; since the system is translationally invariant along the x and y axes, the system is effectively a 1D problem. The TFT junction is thus described by the Hamiltonian $\mathcal{H} = \int dz\, dz'\, \mathcal{H}(z', z)$ where the Hamiltonian density is defined as

$$\mathcal{H}(z',z) = \sum_\sigma \psi^\dagger_\sigma(z')\delta(z'-z)\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} - \mu + U_0(z)\right] \times \psi_\sigma(z) + \tfrac{1}{2}\Delta(z,z')\sum_\sigma\{\sigma e^{-i\sigma\theta_j}\psi^\dagger_\sigma(z')\psi^\dagger_\sigma(z) + \text{H.c.}\} + \mathbf{M}(z,z')\cdot\sum_{\alpha,\beta}\psi^\dagger_\alpha(z')\hat{\boldsymbol{\sigma}}_{\alpha\beta}\psi_\beta(z) \qquad (1)$$

where $\psi^\dagger_\sigma(z)$ and $\psi_\sigma(z)$ are respectively the fermionic creation and annihilation operators for a particle with spin σ at z, $\hat{\sigma}$ are the Pauli matrices and $\Delta(z,z') = -\Delta(z',z)$ is the

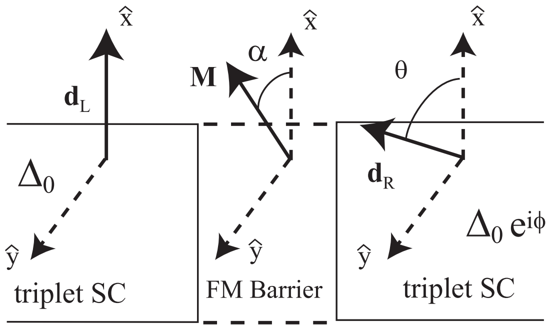

Figure 1. Schematic diagram of the 1D triplet superconductor–ferromagnet–triplet superconductor (TFT) Josephson junction studied in this paper.

p_z-wave superconducting gap. The **d** vector of the triplet superconductors is defined as $\mathbf{d} = (\cos(\theta_j), \sin(\theta_j), 0)$. The subscript j refers to the side of the junction: for $z < 0$ we have $j = L$, whereas for $z > 0$ we have $j = R$. Without loss of generality, we take $\mathbf{d}_L$ to be oriented along the $\hat{\mathbf{x}}$ axis (i.e. $\theta_L = 0$) and the $\mathbf{d}_R$ vector to lie in the spin xy plane (i.e. $\theta_R = \theta$). The charge scattering by the insulating ferromagnetic junction is given by the term $U_0(z) = U_0\delta(z)$ on the first line of (1). The ferromagnetic moment of the junction is defined as $\mathbf{M}(z, z') = (M_\perp \cos(\alpha), M_\perp \sin(\alpha), M_\parallel)\delta(z)\delta(z')$ and the magnetic scattering is described by the last term in (1).

The Hamiltonian (1) is diagonalized by the Bogoliubov–de Gennes (BdG) unitary transformation:

$$\psi_\uparrow(z) = \sum_n \{u_n(z)a_n + v_n(z)b_n^\dagger + w_n(z)b_n + x_n(z)a_n^\dagger\} \quad (2a)$$

$$\psi_\downarrow(z) = \sum_n \{-u_n(z)b_n + v_n(z)a_n^\dagger - w_n(z)a_n + x_n(z)b_n^\dagger\}. \quad (2b)$$

The sum runs over all the eigenstates of the junction. As we are only interested in the localized Andreev states, we drop the subscript n and search for an eigenstate with the wavefunction

$$\Psi_j = \begin{pmatrix} u_j(z) \\ v_j^*(z) \\ w_j(z) \\ x_j^*(z) \end{pmatrix} = e^{c_j \kappa z} \sum_{\gamma=\pm} A_{j,\gamma} \begin{pmatrix} u_{j,\gamma} \\ v_{j,\gamma}^* \\ w_{j,\gamma} \\ x_{j,\gamma}^* \end{pmatrix} e^{\gamma i k_F z} \quad (3)$$

where $c_j = +1(-1)$ for $j = L(R)$, κ^{-1} is the decay length of the Andreev state and the Fermi vector is defined as $k_F = \sqrt{2m\mu}/\hbar$. The elements of $\Psi_j(z)$ are determined by solving the BdG equations:

$$Eu(z) = [T + (U_0 + M_\parallel)\delta(z) - \mu]u(z) - ie^{-i\theta_j}\Delta_j\partial_z v(z) - e^{-i\alpha}M_\perp\delta(z)w(z) \quad (4a)$$

$$Ev(z) = -[T + (U_0 + M_\parallel)\delta(z) - \mu]v(z) - ie^{i\theta_j}\Delta_j^*\partial_z u(z) - e^{i\alpha}M_\perp\delta(z)x(z) \quad (4b)$$

$$Ew(z) = [T + (U_0 - M_\parallel)\delta(z) - \mu]w(z) - ie^{i\theta_j}\Delta_j\partial_z x(z) - e^{i\alpha}M_\perp\delta(z)u(z) \quad (4c)$$

$$Ex(z) = -[T + (U_0 - M_\parallel)\delta(z) - \mu]x(z) - ie^{-i\theta_j}\Delta_j\partial_z w(z) - e^{-i\alpha}M_\perp\delta(z)v(z) \quad (4d)$$

where $T = -\hbar^2\partial_z^2/2m$, $\Delta_L = \Delta_0$ and $\Delta_R = \Delta_0 e^{i\phi}$. The wavefunctions $\Psi_j(z)$ obey the boundary conditions

$$\Psi_L(0^-) = \Psi_R(0^+) \quad (5a)$$

$$\partial_z\Psi_R(0^+) - \partial_z\Psi_L(0^-) = \frac{2m}{\hbar^2}\begin{pmatrix} \hat{Q}_+ & \hat{P} \\ \hat{P}^\dagger & \hat{Q}_- \end{pmatrix}\Psi_R(0^+) \quad (5b)$$

where $\hat{Q}_\pm = (U_0 \pm M_\parallel)\hat{\sigma}_0$ and $\hat{P} = -M_\perp(\hat{\sigma}_3\cos(\alpha) - i\hat{\sigma}_0\sin(\alpha))$.

We obtain two distinct solutions of the BdG equations, with eigenvalues

$$\frac{E_{a(b)}}{k_F\Delta_0} = \frac{1}{2}\sqrt{D}|\sqrt{DA + B} + (-)\sqrt{DA - B}| \quad (6)$$

where

$$A = \{(1 + 2g^2 + g'^2 + Z^2) + (1 + g'^2 + Z^2)\cos(\phi)\cos(\theta) + g^2\cos(\theta - 2\alpha)[\cos(\theta) - \cos(\phi)] - 2Zg'\cos(\theta)\sin(\phi)\} \quad (7a)$$

$$B = 2\cos[(\theta + \phi)/2]\cos[(\theta - \phi)/2] \quad (7b)$$

$$D = [(1 + g^2 + g'^2 - Z^2)^2 + 4Z^2]^{-1/2} \quad (7c)$$

where $g = mM_\perp/\hbar^2k_F$, $g' = mM_\parallel/\hbar^2k_F$ and $Z = mU_0/\hbar^2k_F$. For each solution there is a particle-like branch at $-E_{a,b}$ and a hole-like branch at $E_{a,b}$. The current I_J through these states is given by the expression

$$I_J = -\frac{e}{\hbar}\sum_{i=a,b}\frac{\partial E_i}{\partial\phi}\tanh\left(\frac{E_i}{2k_BT}\right). \quad (8)$$

In general, the contribution to the Josephson current by states outside the gap is negligible [3, 10].

In this paper we focus upon the case where only magnetic scattering happens at the barrier, i.e. $U_0 = 0$. We find that the Andreev state energies have qualitatively different ϕ dependence for the cases when the magnetization is perpendicular and parallel to the z axis. This can be seen in figure 2, where we plot the Andreev energies and I_J as a function of the phase difference ϕ between the left and right superconductors. We first consider the case when $\theta = 0$ (i.e. $\mathbf{d}_L = \mathbf{d}_R$). For $g \neq 0$ and $g' = 0$ the Andreev states are degenerate when $\alpha = (2n-1)\pi/2$ where n is an integer, i.e. the ferromagnetic moment is perpendicular to the two **d** vectors. The Andreev state energies then have the same form as near a potential scattering barrier, $E_{a,b} = k_F\Delta_0\sqrt{D}\cos(\phi/2)$ [6]. The zero-energy level crossings at $\phi = (2n + 1)\pi$ result in jump discontinuities in the current as a function of ϕ (see figure 2(b)). For any other value of α the Andreev states are non-degenerate, but there are level crossings at $\phi = 2n\pi$. Furthermore, the zero-energy level crossings are replaced by level touchings and hence the current is a continuous function of ϕ. The situation for $g = 0$ and $g' \neq 0$ is the same as for a transverse magnetic moment with $\alpha = (2n - 1)\pi/2$: as the barrier moment is perpendicular to the two **d** vectors in both these cases, we have the same form for the Andreev state energies and hence also discontinuous jumps in I_J.

A finite value of θ significantly alters the structure of the Andreev states. For $g \neq 0$ and $g' = 0$ the level crossings of the

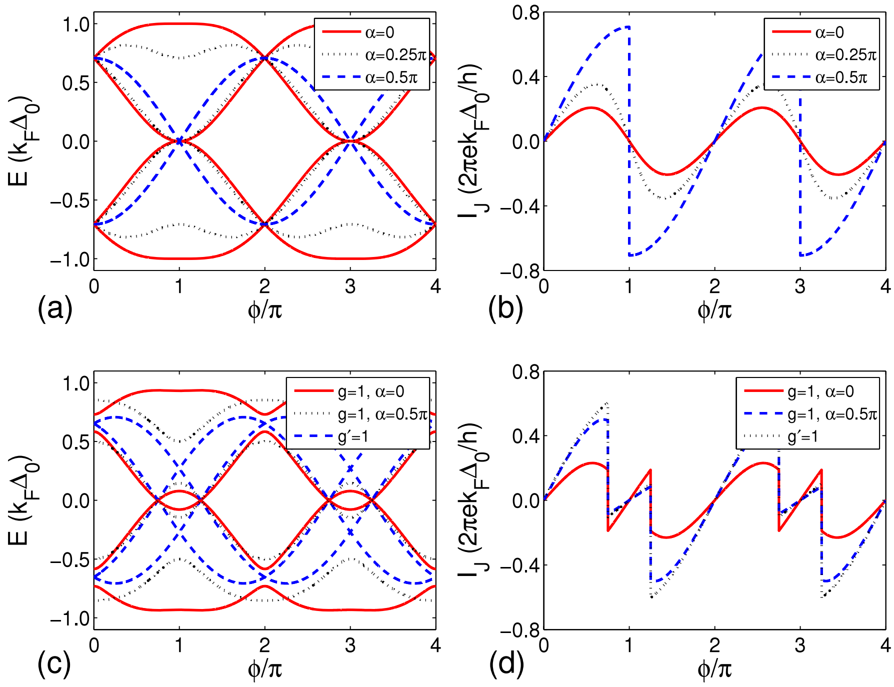

Figure 2. (a) Andreev state energies and (b) I_J as a function of ϕ for $T = 0$, $\theta = 0$, $g' = 0$, $g = 1$ and several values of α. (c) Andreev state energies and (d) I_J as a function of ϕ for $T = 0$, $\theta = 0.25\pi$ and various different barrier parameters. Note that $g \neq 0 \Rightarrow g' = 0$ and vice versa.

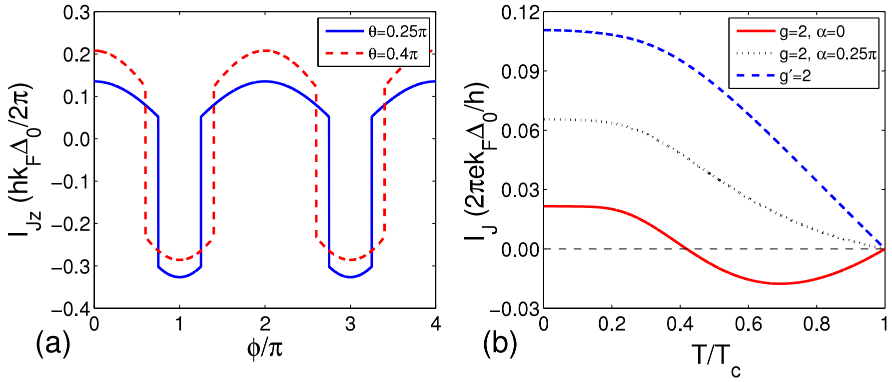

Figure 3. (a) I_{Jz} as a function of ϕ for $g = 0$, $g' = 1$ and several values of θ. (b) Temperature dependence of I_J for $\theta = 0$ and different values of the barrier parameters.

a and b states is removed, while the hole-like and particle-like b states cross at $\phi = (2n - 1)\pi \pm \theta$, as shown in figure 2(c). Note also that the degeneracy of the states at $\alpha = (2n - 1)\pi/2$ is lifted. If we assume that only the states with negative energy are occupied at $T = 0$ (the so-called adiabatic approximation, see [7]), the level crossing causes discontinuous jumps in I_J as the current carried by the b states switches between the particle- and hole-like channels, thus reversing the sign of the current due to the b states. This is demonstrated in figure 2(d). For $g = 0$ and $g' \neq 0$, the effect of $\theta \neq 0$ is quite different: the degeneracy at $\theta = 0$ is lifted by shifting one state by θ to the left and the other state by θ to the right. The a and b states are thus given by $E_{a(b)} = k_F\Delta_0\sqrt{D}\cos((\phi + (-)\theta)/2)$. We note that, for both $g \neq 0$, $g' = 0$ and $g = 0$, $g' \neq 0$ the Josephson current is periodic with period π when $\theta = (2n - 1)\pi/2$ (not shown).

An important physical difference between the cases $g \neq 0$, $g' = 0$ and $g = 0$, $g' \neq 0$ is that in the latter case the spin in the z direction is a good quantum number, whereas in the former the barrier induces mixing between the different spin components. When $g = 0$, therefore, the Andreev states possess well-defined spin quantum numbers: $\sigma = \uparrow$ for state a and $\sigma = \downarrow$ for state b. This has the interesting implication that the transport through the mid-gap states can be spin-polarized. Without a potential term U_0, this is only possible when $\theta \neq 2n\pi$ as here the two Andreev states are non-degenerate because the non-zero θ creates a different phase gradient in the two spin channels [6, 11, 12]. In figure 3(a) we plot the spin projection of the current along the z axis I_{Jz}, where I_{Jz} is defined:

$$I_{Jz} = -\frac{1}{2e}(I_{J\uparrow} - I_{J\downarrow}) \tag{9}$$

and $I_{J\sigma}$ is the spin-σ component of the current. We observe discontinuous reversals of the spin current occurring at the zero crossings of the Andreev bands: the current jumps in figure 2(d) therefore also correspond to changes in the spin polarization. It is interesting to note that, when $U_0 = 0$, the sign of $M_\parallel$ is of no importance in determining the spin current. Because the spin states are mixed by the presence of a transverse component of the magnetic moment, the effect of $g \neq 0$ on the spin polarization of the current is a considerably more difficult problem which will be addressed elsewhere.

The splitting of the Andreev state energies by the transverse magnetic moment can result in an unconventional temperature dependence of I_J, i.e. a reversal of the current as temperature is increased. Assuming a BCS temperature dependence of Δ_0, we plot I_J in figure 3(b) for fixed $\phi = 0.5\pi$ and $\theta = 0$. To understand the origin of this effect, we must consider the change in the populations of the Andreev states with increasing T. At any $T \neq 0$ the positive-energy Andreev states have a non-zero population, although the occupancy of the higher-energy $|E_a|$ state is always lower than the occupancy of the $|E_b|$ state. As the temperature is increased the reduction in the current due to the a states (I_J^a) is therefore less than that due to the b states (I_J^b). Assuming that $|I_J^b| > |I_J^a|$ at $T = 0$ and the sign of $\partial E_a/\partial\phi$ is opposite to $\partial E_b/\partial\phi$, we find that, above some critical temperature, $|I_J^b| < |I_J^a|$ and the current reverses. Because the ϕ dependence of the Andreev states is crucial for the temperature-dependent reversal of I_J, this effect depends sensitively upon the barrier parameters. In particular, the Andreev states converge together as $\alpha \to (2n-1)\pi/2$ (see figure 2(a)) and so the sign change only occurs for α sufficiently close to $n\pi$ and does not occur at all for $g = 0$, $g' \neq 0$. Furthermore, for any fixed ϕ the sign change only occurs for sufficiently large g. Recently a similar temperature-dependent current reversal for a singlet superconductor–ferromagnet–singlet superconductor has been reported [13].

To conclude, we have investigated an unconventional TFT Josephson junction in which I_J displays a complicated dependence upon the orientations of the magnetic moment and $\mathbf{d}_{L,R}$. A temperature-dependent reversal of I_J is predicted, as well as the possibility of a spin-polarized current. These results are identifying signatures of the formation of localized mid-gap Andreev states at the junction barrier.

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